THOUGHTS ON DUALITY AND FUNDAMENTAL CONSTANTS

J. A. Nieto [1], L. Ruiz and J. Silvas

Facultad de Ciencias Físico-Matemáticas, Universidad Autónoma de Sinaloa, C.P. 80000, Culiacán, Sinaloa, México

Abstract

We consider some fundamental constants from the point of view of the duality symmetry. Our analysis of duality is focused on three issues: the maximum radiated power of gravitational waves, the cosmological constant, and the magnetic monopole mass. We show that the maximum radiated power of gravitational waves implies that the Planck time is a minimal time. Furthermore, we prove that duality implies a quantization of the cosmological constant. Finally, by using one of the Euler series for the number π, we show that the Dirac electric-magnetic charge quantization implies a mass for the magnetic monopole (or neutrino) of the order of 10^{-5} the mass of the electron.

Resumen

Consideramos algunas constantes fundamentales desde el punto de vista de la simetría de dualidad. Nuestro análisis de dualidad se enfoca en tres temas: la potencia máxima radiada de ondas gravitacionales, la constante cosmológica y la masa del monopolo magnético. Demostramos que la potencia máxima radiada de ondas gravitacionales implica que el tiempo de Planck corresponde a un tiempo mínimo. Más aun, probamos que la dualidad implica una cuantización de la constante cosmológica. Finalmente, usando una de las series de Euler para el número π, demostramos que la cuantización de la carga eléctrica-magnética de Dirac implica una masa para el monopolo magnético (o neutrino) del orden de 10^{-5} la masa del electrón.

Keywords: Fundamental constants; gravitational waves; cosmological constant; duality.

Pacs numbers: 04.60.-m, 04.30.-w, 6.20.Jr, 98.80.-k

September, 2006

[1] nieto@uas.uasnet.mx

1.- Introduction

Because the problem of the number of fundamental constants [1] and their possible time variability [2] is of permanent interest in physics, any consistent new idea on this subject must be welcome. In this context, it has been emphasized [3] that one should only consider as physically meaningful the variability of dimensionless constants rather than dimensional constants [2]. This claim is not shared, however, by some physicists (see Ref. 2 for details), and therefore new routes for approaching the subject seem to be needed.

One of our aims in this paper is to shed some light on the above controversy by applying the duality concept to some fundamental constants. Specifically, in this work, we analyze some fundamental constants from the point of view of a duality symmetry, including the Planck time, the cosmological constant, and the magnetic monopole mass. We show that by applying the duality concept to the maximum radiated power of gravitational waves one obtains the result that the Planck time must be a minimal time. Furthermore, using the S-duality concept for the cosmological constant, obtained in the linearized gravity development [4], and relaying on analogy of the Dirac's quantization of the electric and magnetic monopole charges, we argue that duality implies a quantization of the cosmological constant. Finally, by using one of the Euler series for the number π, we demonstrate that the Dirac duality concept for the electric charge implies a relation between the electron mass m_e and the magnetic monopole mass m_g. Such a relation leads to a value for m_g of the order of the neutrino mass $\sim 10^{-5}\ m_e$, which is too low in comparison with the expected standard value for the mass of the magnetic monopole, namely of the order of $GeVs$. Thus, we find that duality seems to imply a deep connection between the neutrino $\bar{\nu}_e$ and the magnetic monopole.

Moreover, we explain that the three different types of results mentioned above can be written in a dimensionless constant context. This suggests that the underlying theory must be invariant under the duality of the dimensionless fundamental constants rather than a duality of dimensional constants. This result is in agreement with Dirac's older idea [3] (see Ref. 2 for a recent discussion of this problem) that dimensionless constants are more important than dimensional ones. From this perspective, one may conclude from our results that in fact what matters is the variability of dimensionless funda-

mental constants, as Duff has emphasized [2], rather than the variability of dimensional fundamental constants.

This article is organized as follows. In Sec. 2, using the maximum radiated power of gravitational waves, we prove that the Planck time is a minimal time. In Sec. 3, we discuss the cosmological constant duality, and in Sec. 4 we analyze the magnetic monopole mass from a duality perspective. Finally, in Sec. 5, we make some latter remarks.

2.- Duality between the maximum radiated power and Planck time

Consider a source of gravitational waves of mass M and radius r. It is known that an estimate of the radiated power of gravitational waves is given by

$$P \sim L_0 (\frac{r_{Sch}}{r})^5, \tag{1}$$

where

$$L_0 = \frac{c^5}{G}, \tag{2}$$

and

$$r_{Sch} = \frac{GM}{c^2}. \tag{3}$$

Here, c is the "light" velocity (or spacetime structure constant in the terminology of Ref. 5) and G is the Newton gravitational constant. In order to avoid the collapse of the object into a black hole, it is necessary to have $r_{Sch} < r$ and therefore from formula (1) we see that the maximum radiated power of any object is L_0. Conversely, if we assume that L_0 is the maximum radiated power, then from (1) we obtain the relation $r_{Sch} < R$, which is linked to the relation $v < c$, where v is the velocity of the source.

Let us now introduce the Planck time

$$t_P = (\frac{G\hbar}{c^5})^{1/2}, \tag{4}$$

where $\hbar$ is the Planck constant. This formula can be written as

$$\frac{\hbar}{t_P^2} = \frac{c^5}{G} = L_0. \tag{5}$$

Therefore, by fixing $\hbar$, we obtain the interesting dual property: L_0 is the maximum radiated power if and only if t_P is a minimal time. Of course, when c is setting, one has that minimal time t_P implies that the Planck length $l_P = ct_P = (\frac{G\hbar}{c^3})^{1/2}$ is a minimum length in nature (see Ref. 6). Although, this result seems to be in agreement with the idea that a fundamental length arose in the string theory (see Ref. 7), its classical derivation presented here contrasts with the same result obtained from quantum gravity (see Refs. 8 to 11, and references. therein).

3.- Cosmological constant duality

In Ref. 4 it was proved that linearized gravity *a la* MacDowell-Mansouri implies a cosmological constant duality symmetry

$$\Lambda \leftrightarrow \frac{1}{\Lambda}, \tag{6}$$

which can be thought as the analogue of the charge duality in an Abelian gauge field theory,

$$e^2 \leftrightarrow \frac{1}{e^2}. \tag{7}$$

In order to clarify this analogy, let us briefly describe the main result of Ref. 4. Let us introduce the 'gauge' field of linearized gravity,

$$A_{\mu\alpha\beta} = \frac{1}{2}(\partial_\alpha h_{\mu\beta} - \partial_\beta h_{\mu\alpha}) = -A_{\mu\beta\alpha}. \tag{8}$$

Under the transformation

$$\delta A_{\mu\alpha\beta} = \partial_\mu \lambda_{\alpha\beta}, \tag{9}$$

the curvature tensor

$$F^{\alpha\beta}_{\mu\nu} = \partial_\mu A^{\alpha\beta}_\nu - \partial_\nu A^{\alpha\beta}_\mu \tag{10}$$

is invariant. This means that the tensor $F^{\alpha\beta}_{\mu\nu}$ can be identified with an abelian field strength.

Consider the extended curvature

$$\mathcal{F}^{\alpha\beta}_{\mu\nu} = F^{\alpha\beta}_{\mu\nu} + \Omega^{\alpha\beta}_{\mu\nu}, \tag{11}$$

where

$$\Omega^{\alpha\beta}_{\mu\nu} = \delta^{\alpha}_{\mu} h^{\beta}_{\nu} - \delta^{\beta}_{\mu} h^{\alpha}_{\nu} - \delta^{\alpha}_{\nu} h^{\beta}_{\mu} + \delta^{\beta}_{\nu} h^{\alpha}_{\mu}. \tag{12}$$

In Ref. 4 it was shown that the action

$$\mathcal{S} = \frac{1}{16\Lambda}\int d^4x \epsilon^{\mu\nu\alpha\beta}\mathcal{F}^{\tau\lambda}_{\mu\nu}\mathcal{F}^{\sigma\rho}_{\alpha\beta}\epsilon_{\tau\lambda\sigma\rho} + \frac{i\Theta}{8\pi}\int d^4x \epsilon^{\mu\nu\alpha\beta}\mathcal{F}^{\tau\lambda}_{\mu\nu}\mathcal{F}^{\sigma\rho}_{\alpha\beta}\delta_{\tau\lambda\sigma\rho}, \tag{13}$$

where Λ and Θ are constants, permits a dual action. From (13) we observe that the cosmological constant Λ is playing the role of a gauge coupling constant g^2, and that Θ is playing the role of a θ constant in the usual abelian Maxwell theory. Thus, we find that the analogue of the gauge coupling constant duality $g^2 \to \frac{1}{g^2}$ in the case of linearized gravity corresponds to the cosmological constant duality transformation $\Lambda \to \frac{1}{\Lambda}$ (see Ref. 4 for details).

In this section we are interested in a deep understanding of the relation (6). For this purpose let us recall how the relation (7) arises in an Abelian gauge field theory. It turns out that the origin of (7) is the Dirac's electric charge quantization condition, namely

$$ge = \frac{n\hbar c}{2}, \tag{14}$$

where g is the magnetic monopole charge. The key point is that the source-free Maxwell field equations are invariant under the transformation

$$\begin{aligned} E &\to B \\ B &\to -E. \end{aligned} \tag{15}$$

While in the case of nonsource-free Maxwell equations the transformation (9) needs to be extended and accompanied by the transformation

$$g \leftrightarrow e. \tag{16}$$

Due to (14), one sees that (16) is equivalent to (7).

In general, the cosmological constant Λ can be written in terms of a fundamental length l in the form

$$\Lambda = \pm\frac{(D-1)(D-2)}{2l^2}, \tag{17}$$

where D is the dimension of the spacetime of an arbitrary signature. Therefore, the duality relation (6) is equivalent to

$$l^2 \leftrightarrow \frac{1}{l^2}. \tag{18}$$

We observe that (18) establishes the analogy between (6) and (7) in a clearer context. Thus, following this analogy, one should expect (18) to be a consequence of the quantization relation

$$\mathcal{L}l = \frac{nl_pR}{2}, \tag{19}$$

where l_p is the Planck length, R is the radius of the universe and $\mathcal{L}$ is the dual length associated with l. In turn, this result implies a quantization of l, and therefore a quantization of the cosmological constant via the relation (17). In fact, by writing $\Lambda_l \equiv \Lambda$ and

$$\Lambda_{\mathcal{L}} = \pm\frac{(D-1)(D-2)}{2\mathcal{L}^2}, \tag{20}$$

we discover that (19) implies the formula

$$\Lambda_{\mathcal{L}}\Lambda_l = \frac{(D-1)^2(D-2)^2}{n^2l_p^2R^2}. \tag{21}$$

Of course, the cases $D = 1$ and $D = 2$ are exceptional, as can be seen even from (17). So, out of these two cases, one may be interested in an understanding of the meaning of (19) and (21). First of all, if $\Lambda_{\mathcal{L}} \neq 0$, we discover that Λ_l should be quantized. Second, assuming $\mathcal{L} \sim \frac{R}{2}$, we observe from (19) that $l = nl_p$ and therefore l_p is a minimal length, in agreement with our discussion in Sec. 3. Finally, from (19) we see that, taking $\mathcal{L} \sim \frac{l_p}{2}$, one obtains $l = nR$, and therefore from (17) or (21) we find that

$$\Lambda_l = \pm\frac{(D-1)(D-2)}{2n^2R^2}. \tag{22}$$

For $n = 1, D = 4$ and $R \sim 10^{28}cm$ we get $\Lambda_l \sim 10^{-56}cm^{-2}$, which is a very small value but nevertheless different from zero. It is not difficult to see that these results can be dualized, that is, when Λ_l is small, $\Lambda_{\mathcal{L}}$ is large and vice versa. For historical reasons the attempt to make zero the cosmological constant is called "the cosmological constant problem". From (21) we observe

that for $D \neq 1$ and $D \neq 2$, this type of problem has no a solution free of singularities. In fact, (21) implies that if $\Lambda_l \to 0$, then $\Lambda_{\mathcal{L}} \to \infty$ and vice versa.

4.- The magnetic monopole mass duality

Consider the duality transformations

$$g^2 \longleftrightarrow \frac{1}{e^2} \tag{23}$$

and

$$m_g \longleftrightarrow \frac{1}{m_e}. \tag{24}$$

Observe that (23) is a consequence of (14). In (24), m_g refers to the mass of the magnetic monopole. Moreover, we are assuming that there exist the analogue of the formula (24) for the mass quantization as Zee [12] has suggested for any massive system. It is not difficult to see that the relation

$$\frac{m_g g^2}{m_e e^2} = \beta, \tag{25}$$

is invariant under the transformations (23) and (24). Thus, the constant β in (25) must be fundamental, dimensionless, and should not be related to any property of the system. On the other hand, it is known that not only the fine structure constant $\alpha = \frac{e^2}{\hbar c}$ can be related to the number π via the Weyler heuristic formula

$$\alpha = \frac{9}{8\pi^4}\left(\frac{\pi^5}{2^4 5!}\right)^{1/4}, \tag{26}$$

but also all masses of fundamental particles via the hiperdiamons lattices based on Clifford algebras (see Ref. 13 and references therein). This suggests that β in (25) could, in principle, be related to the number π. Let us choose one of the simplest possibilities for such a constant, namely $\beta = a\pi^2$, where a is a numerical factor independent of π to be determined below. Thus expression (25) becomes

$$\frac{m_g g^2}{m_e e^2} = a\pi^2. \tag{27}$$

Using (14) and the fine structure constant $\alpha = \frac{e^2}{\hbar c}$, formula (27) yields

$$m_g = 4am_e\alpha^2\pi^2. \tag{28}$$

It turns out to be convenient to multiply this expression by c^2

$$m_g c^2 = 4am_e c^2\alpha^2\pi^2. \tag{29}$$

On the other hand, there exists a famous numerical series due to Euler for determining the number π, namely

$$\sum_{n=1}^{\infty} \frac{1}{n^2} = \frac{\pi^2}{6}. \tag{30}$$

which can be used in the Eq. (29) to obtain the intriguing result

$$m_g c^2 = \sum_{n=1}^{\infty} \frac{m_e c^2 \alpha^2}{2} \frac{1}{n^2}, \tag{31}$$

provided we set $a = \frac{1}{2(4!)}$. Therefore, we have shown that using (14) the invariant formula (25) with $\beta = \frac{\pi^2}{2(4!)}$ leads to (31). We recognize in the expression

$$E_n \equiv -\frac{m_e c^2 \alpha^2}{2} \frac{1}{n^2} \tag{32}$$

the well known formula for the eingenvalues of the energy for the hydrogen atom. From (31) we find that the value of m_g is of the order of the neutrino mass, $m_{\nu_e} \sim 10^{-5} m_e$, but too low in comparison with the expected standard value for the magnetic monopole mass, which is of the order of $GeVs$. One may try to understand this result by considering the well known neutron decay

$$n \to p + e + \bar{\nu}_e. \tag{33}$$

A hydrogen atom is made out of a proton p and an electron e. Thus, the transition (33) suggests that the total energy obtained by the eigenvalues of the energy according to (32) should determine the mass of the neutrino $\bar{\nu}_e$. However, relation (32) suggests identifying m_v with m_g, and therefore, we

may conclude that duality seems to imply a deep connection between the neutrino $\bar{\nu}_e$ and the magnetic monopole.

5.- Final Remarks

In this work we have shown that duality at the level of fundamental constants leads to some interesting and intriguing conclusions: the Planck time is a minimal time, the cosmological constant is quantized and the magnetic monopole mass is related to the neutrino mass. One should expect similar observations if the duality concept is applied to other physical scenarios.

A question arises whether this duality of the fundamental constants might shed some light on the controversy about the variability of fundamental constants. Let us write formula (19) (for $n = 1$) as

$$\frac{\mathcal{L}}{R}\frac{l}{l_p} = \frac{1}{2}. \tag{34}$$

We observe that this is a duality relation between two dimensionless constants $\frac{\mathcal{L}}{R}$ and $\frac{l}{l_p}$. Similarly, considering the ratios $\frac{m_g}{m_e}$ and $\frac{g^2}{e^2}$, one sees that (25) is a duality expression between two dimensionless constants. Of course, exactly the same conclusion can be obtained from the Dirac's quantization condition (14), since in that case one may write (for $n = 1$)

$$\frac{g^2}{\hbar c}\frac{e^2}{\hbar c} = \frac{1}{4}. \tag{35}$$

These observations mean that, from the point of view of duality symmetry what seems to be essential are the dimensionless constants rather than the dimensional ones, in agreement with Dirac's argument [3] and Duff's reply [2]. In fact, it is easy to see that duality in terms of fundamental dimensional constants does not make sense. For instance, let us assume a duality for the light velocity c of the form

$$c^2 \leftrightarrow \frac{1}{c^2}. \tag{36}$$

If we set $c = 1$ then this symmetry is lost. Thus, in order to maintain the duality symmetry of an underlaying theory, it is necessary to express it in terms of dimensionless constants. In turn, this implies that what matters is

the variability of such dimensionless constants, rather than dimensional constants. Considering this observation, we discover that (34) and (35) establish that time variability of a dimensionless fundamental constant implies a time variability of its corresponding dual.

Now, one should expect that the duality of the dimensionless fundamental constants is reestablished in a duality at the level of the fundamental field theory. Maxwell field theory, with both electric and magnetic sources, offers an excellent example of this remark. Therefore, one should be interested in applying the ideas discussed in this paper in a corresponding field theory in which duality may play a fundamental role. In fact, the duality for linearized gravity used in Sec. 3 as starting point in connection with the duality of the cosmological constant is a good example of this idea. However, one may still be more ambitious and ask for a theory in which duality acts as a fundamental principle. In a sense, this is the principle suggested by the interconnection between the various string theories leading to the so-called M-theory [14]. Thus, one may say that M-theory is the final goal of a duality principle. The fine point is that this idea may require a new and unexpected mathematical framework for its realization. In a series of works [15]-[22], it has become more evident that a candidate for such a mathematical framework is the oriented matroid theory [23]. Hence, one of our aims for further research is to use the oriented matroid theory as a mathematical tool in order to have a better understanding of the duality of fundamental constants.

The main idea of the present work was to link duality symmetry with various fundamental constants. In this respect, it is worth mentioning that a relation between the cosmological constant and atomic units has been established a long time ago [24]. In fact, this relation seems to present some kind of duality between the cosmological constant similar to the present discussion. Therefore, it may be interesting for further research to analyze the ideas of Ref. 24 from the point of view of the present work. Furthermore, there will be effects of duality symmetry in connection with fundamental constants, and in particular with the cosmological constant, which we might hope to be able to measure. In this sense the cosmic geophysical observations discussed in Ref. 25 may be a guide, and this is something we hope to consider in the near future.

From the present work the following natural questions may emerge:

(i) The expression (1) for the radiated power of gravitational waves is calculated in linearized GRT, i.e., for weak gravitational fields. What sense does it make then to bring it into context with the Planck time which governs

extremely strong gravity?

(ii) What does it mean to quantize a fundamental constant, as motivated by some formal analogy for the cosmological constant? Wouldn't it be a proposal against the spirit of such a constant?

(iii) Is there any physical meaning of the sum over all infinite energy levels of the Hydrogen atom?

It is clear that, although these questions are interesting, their answer might not be so simple. Nevertheless, it is tempting to try giving a possible answer. Let us first discuss the question (i). It turns out that exactly the same question can arise in the case of weak/strong coupling duality of linearized gravity [4,26]. The answer in this case may rely on the assumption of dual 'phases' of M-theory: one which describes weak gravity and the other, strong gravity. And each one would have their own field theory limit. But the idea is that the M-theory itself becomes invariant under a weak-strong duality transformation. From this perspective, it seems surprising that one may touch this idea of dual phases of M-theory by simply considering the duality between the maximum radiated power of gravitational waves and Planck time. A similar argument can be applied in the case of question (ii). M-theory should have two dual phases each one with small/large cosmological constant. So, the traditional spirit of the cosmological constant comes from just one of these dual phases, but as soon as one realizes the possibility of the other dual gravitational phase then the quantization of the cosmological constant becomes as a consequence. It is worth mentioning that the idea of the quantum cosmological constant has already appeared in other contexts [27,28]. At first sight it seems that the question (iii) should correspond to a different scenario. However, since we have assumed in Sec. 4 the weak/strong coupling duality for an Abelian gauge theory, which is presumably part of M-theory, we find that a possible answer might also be found in the concept of dual phases of M-theory. In fact, suppose that we have a system in which in one phase can be described by the associated constants m_e and e and in the other by m_g and g, respectively. In order for this description to make sense, something must remain constant. According to formula (25) this is provided by the combinations $m_g g^2$ and $m_e e^2$. Thus, such a constant must be fundamental, dimensionless, and should not be related to any property of the system itself. What other than the number π? It just happens that, as the Weyler heuristic formula and the formula (27) indicates, such a constant should be proportional to π^2 rather than π itself. Now, from (27), one may obtain (29). The next step is simply to apply the famous numerical series

(30) due to Euler for determining the number π^2. What we obtain is the energy formula (31), which can be related to the hydrogen atom. From this perspective, one has obtained the surprising result that the quantum energy formula for the hydrogen atom is a consequence of the dual phases of M-theory.

Although the above explanations in terms of the M-theory seem reasonable, one can still have the feeling that the questions above require further discussion. For instance, M-theory does not give an answer to the question: What is the strong gravitational coupling phase? Attempts to answer this question have been given by Nieto [4] and Hull [26]. In particular, Hull's idea is to construct a theory from the dual gauge fields

$$D_{\mu\nu\alpha} = \epsilon_{\mu\nu\alpha\beta} h^{\beta}_{\alpha} \tag{37}$$

and

$$C_{\mu\nu\alpha\gamma\rho\sigma} = \epsilon_{\mu\nu\alpha\beta}\epsilon_{\gamma\rho\sigma\lambda} h^{\beta\lambda}, \tag{38}$$

which are duals of the gravitational fluctuation h. Although these ideas have generated some motivation (see Ref. 29 and references therein), complete dual gravitational theory is still a mystery. Thus, since the strong gravitational coupling phase is an open problem, one cannot expect to give a general answer at the present to the above questions in terms of the M-theory.

Acknowledgment: This work was supported in part by the UAS under the program PROFAPI-2006.

References

[1] M. J. Duff, L. B. Okun and G. Veneziano, JHEP **0203**, 023 (2002); physics/0110060.

[2] M. J. Duff, "Comments on Time Variation of Fundamental Constants", hep-th/0208093.

[3] P. A. M. Dirac, Nature **139** (1937).

[4] J. A. Nieto, Phys. Lett. A **262**, 274 (1999); hep-th/9910049.

[5] G. F. R. Ellis and J. P. Uzan, Am. J. Phys. **73**, 240 (2005); gr-qc/0305099.

[6] G. Veneziano, "Physics with a Fundamental Length", in Memorial Volume dedicated to Dima Knizhnik.

In *Brink, L. (ed.) et al.: Physics and mathematics of strings* 509-529 and CERN Geneva - TH. 5581 (89,rec.Jan.90) 20 p.

[7] T. Padmanabhan, Gen. Rel. Grav. **17**, 215 (1985); see also W. M. Saslow, Eur. J. Phys. **19**, 313 (1998).

[8] B. S. DeWitt, Gravitation, *An introduction to current research*, ed. by L. Witten (Wiley, New York, 1962).

[9] L. Rosenfeld, *Selected papers of León Rosenfeld*, ed. by R. S. Cohen and J. J. Stachel (Reidel, Dordrecht, 1966).

[10] E. Prugovecki, Found. Phys. **26**, 1645 (1996); gr-qc/9603044, see also H. H. von Borzeskowski and H. J. Treder, *The Meaning of Quantum Gravity* (Reidel, Dordrecht, 1988).

[11] L. J. Garay, Int. J. Mod. Phys. A**10**, 145 (1995); gr-qc/9403008.

[12] A. Zee, Phys. Rev. Lett. **55**, 2379 (1985); Erratum-ibid. **56**, 1101 (1986).

[13] C. Castro, Found. Phys. **34,** 1091 (2004).

[14] M. J. Duff, "Progress in M-Theory", Prepared for 23rd Annual MRST (Montreal-Rochester-Syracuse-Toronto) Conference on High-Energy Physics (MRST 2001), London, Ontario, Canada, 16-18 May 2001. Published in *London 2001, Theoretical high energy physics* 3-18 Also in *Fort Lauderdale 2001, Cosmology and elementary particle physics* 245-254 Also in *Cairo 2001, High energy physics* 103-117 M-theory.

[15] J. A. Nieto, Rev. Mex. Fis. **44**, 358 (1998); hep-th/9807107.

[16] J. A. Nieto and M. C. Marín, J. Math. Phys. **41**, 7997 (2000); hep-th/0005117.

[17] J. A. Nieto, J. Math. Phys. **45**, 285 (2004); hep-th/0212100.

[18] J. A. Nieto and M.C. Marín, Int. J. Mod. Phys. A**18**, 5261 (2003); hep-th/0302193.

[19] J. A. Nieto, Adv. Theor. Math. Phys. **8**, 177 (2004); hep-th/0310071.

[20] J. A. Nieto, Rev. Mex. Fis. E **51**, 5 (2005); hep-th/0407093.

[21] J. A. Nieto, "Oriented Matroid Theory as a Mathematical Framework for the M-theory", hep-th/0506106.

[22] J. A. Nieto, "Toward a Connection Between the Oriented Matroid Theory and Supersymmetry", hep-th/0510185.

[23] A. Björner, M. Las Vergnas, B. Sturmfels, N. White and G. M. Ziegler, *Oriented Martroids* (Cambridge University Press, Cambridge, 1993).

[24] H. Ertel, Sitzungsberichte Preussische Akademie der Wissenschaften, Phys. Math. Klasse, **1**, 3 (1935); Naturwissenschaften **23,** 36 (1935); Naturwissenschaften, **23,** 70 (1935); Physikalische Zeitschrift **37**, 138 (1936).

[25] W. Schröder and H. J. Treder, Found. Phys. **28**, 1013 (1998).

[26] C. M. Hull, Nucl. Phys. B **583**, 237 (2000); hep-th/0004195.

[27] I. Moss, Phys. Lett. B **283**, 52 (1992).

[28] A. Pinzul and A. Stern, Class. Quant. Grav. **23**, 1009 (2006); hep-th/0511071.

[29] H. Casini, R. Montemayor and L. F. Urrutia, Phys. Rev. D **68**, 065011 (2003); hep-th/0304228.